

Constraints on scalar-tensor models of dark energy from observational and local gravity tests

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We construct a family of viable scalar-tensor models of dark energy (DE) which possess a phase of late-time acceleration preceded by a standard matter era, while at the same time satisfying the local gravity constraints (LGC). The coupling Q between the scalar field and the non-relativistic matter in the Einstein frame is assumed to be constant in our scenario, which is a generalization of $f(R)$ gravity theories corresponding to the coupling $Q = -1/\sqrt{6}$. We find that these models can be made compatible with local gravity constraints even when $|Q|$ is of the order of unity through a chameleon mechanism, if the scalar-field potential is chosen to have a sufficiently large mass in the high-curvature regions. We show that these models generally lead to the divergence of the equation of state of DE, which occur at smaller redshifts as the deviation from the Λ CDM model become more significant. We also study the evolution of matter density perturbations and employ them to place bounds on the coupling $|Q|$ as well as model parameters of the field potential from observations of the matter power spectrum and the CMB anisotropies. We find that, as long as $|Q|$ is smaller than the order of unity, there exist allowed parameter regions that are consistent with both observational and local gravity constraints.

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I. INTRODUCTION

The origin of dark energy (DE) has persistently posed one of the most serious mysteries in modern cosmology [1, 2]. The first step toward understanding the nature of DE is to clarify whether it is a simple cosmological constant or it originates from other sources that dynamically change in time. The dynamical DE models can be distinguished from cosmological constant by studying the variation of the equation of state of DE ($= w_{\text{DE}}$) as well as the evolution of density perturbations. The scalar field models of DE such as quintessence [3] and k-essence [4] predict a wide variety of variations in w_{DE} , but still the current observational data are not sufficient to rule out such models unless the equation of state shows a peculiar evolution. Moreover, the scalar field is required to have a light mass m_ϕ comparable to the present Hubble parameter ($m_\phi \sim 10^{-33}$ eV), in order to give rise to an accelerated expansion. This requirement is generally difficult to reconcile with fifth-force experiments unless there exists some mechanism by which the interaction range of the scalar-field mediated force can be made shorter.

There exists another class of dynamical DE models that modify Einstein gravity. The simplest models that belong to this class are those that are based on the so called $f(R)$ gravity theories in which the Lagrangian density f is a function of the Ricci scalar R . It is well known that theories of the type $f(R) = R + \alpha R^2$ can give rise to an inflationary expansion in the early universe because of the dominance of the αR^2 term [5]. In the context of DE, the model $f(R) = R - \mu^{2(n+1)}/R^n$ ($n > 0$) was proposed to explain the late-time accelerated expansion due to the dominance of the term $\mu^{2(n+1)}/R^n$ [6] (see also Refs. [7]). It was found, however, that this model is plagued by a number of problems such as the instability of matter perturbations [8] as well as the absence of a matter-dominated epoch [9].

In the past few years there has been a burst of activity in the search for viable $f(R)$ DE models [10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. In Ref. [11] the conditions for the cosmological viabilities of $f(R)$ DE models (having a matter era

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followed by an accelerated epoch) were derived without specifying the form of $f(R)$. A number of general conditions are required on general viability and stability grounds. For the existence of a prolonged saddle matter era, the quantity $m = Rf_{,RR}/f_{,R}$ needs to be positive and close to 0. To avoid anti-gravity, $f_{,R}$ is required to be positive in regions $R \geq R_1$, where $R_1 (> 0)$ is a Ricci scalar at a de-Sitter attractor responsible for the accelerated expansion. Also, to ensure that density perturbations do not exhibit violent instabilities, we require $f_{,RR} > 0$ [13, 14]. The conditions $f_{,RR} > 0$ and $f_{,R} > 0$ (for $R > R_1$) have been shown to also ensure the absence of ghosts and tachyons [15].

The local gravity constraints (LGC) should also be satisfied for the viability of $f(R)$ models [20]. The $f(R)$ gravity in the metric formalism is equivalent to scalar-tensor theory with no scalar kinetic term, namely, the Brans-Dicke model with a potential and $\omega_{\text{BD}} = 0$ [21]. If the mass of the scalar-field degree of freedom always remains as light as the present Hubble parameter H_0 , one can not satisfy the LGC due to the appearance of the long-ranged fifth force. It is possible to design the field potential so that the mass of the field is heavy in a large-curvature region where local gravity experiments are carried out. Then the interaction range of the fifth force becomes short in such a high-density region, which allows the possibility of the models being compatible with LGC.

In fact a number of viable models based on $f(R)$ theories have been proposed [15, 16, 17, 18] that can satisfy both the cosmological and local gravity constraints discussed above. In the high-density region ($R \gg R_c$) these models have asymptotic behavior $f(R) \simeq R - \mu R_c [1 - (R/R_c)^{-2n}]$ ($\mu > 0$, $R_c > 0$, $n > 0$), where R_c is of the order of the present Ricci scalar. Inside a spherically symmetric body with an energy density ρ_m , the field acquires a minimum at $R \simeq \rho_m$ with a mass much heavier than H_0 . In this case the body has a thin-shell inside it so that the effective coupling between the field and the matter decreases through the so-called chameleon mechanism [12, 22, 23, 24, 25]. The bounds on the model parameters of such models derived from solar-system and equivalence principle constraints are given by $n > 0.5$ and $n > 0.9$, respectively [25]. For viable $f(R)$ models there are also a number of interesting observational signatures such as the divergence of the equation of state of DE [18, 26] and the peculiar evolution of matter perturbations [15, 16, 18, 19]. This is useful to distinguish $f(R)$ gravity models from the Λ CDM model.

In the Einstein frame the $f(R)$ gravity corresponds to a constant coupling $Q = -1/\sqrt{6}$ between dark energy and the non-relativistic fluid [9] (see Eq. (10) for the definition of Q). Basically, this is equivalent to the coupled quintessence scenario [27] with a specific coupling. Our aim in this paper is to generalize the analysis to scalar-tensor theories with the action (13) in which the coupling Q is an arbitrary constant. We regard the Jordan frame as a physical one in which the usual matter conservation law holds. The dark energy dynamics in scalar-tensor theories has been investigated in many papers [28, 29, 30, 31, 32] after the pioneering works of Refs. [33, 34]. If the mass of the field ϕ is always of the order of H_0 , the solar-system constraint $\omega_{\text{BD}} > 4.0 \times 10^4$ [35] gives the bound $|Q| < 2.5 \times 10^{-3}$. Previous studies dealing with the compatibility of the scalar-tensor DE models with LGC have restricted their analysis to this small coupling region [30, 31]. We wish to extend the analysis to the case in which the coupling $|Q|$ is larger than the above massless bound. In fact one can design the potential $V(\phi)$ so that the mass of the field is sufficiently heavy in the high-density region to satisfy LGC through the chameleon mechanism. We shall construct such a viable field potential inspired by the case of the $f(R)$ gravity and place experimental bounds on model parameters in terms of the function of Q .

We shall also study the variation of the equation of state for DE and the evolution of density perturbations in such scalar-tensor theories. Interestingly, we find that the divergent behavior of w_{DE} is also present as in the case of $f(R)$ gravity. We also estimate the growth rate of matter perturbations and show that the non-standard evolution of perturbations manifests itself from a certain epoch (depending upon model parameters) during the matter era. This is useful to place constraints on model parameters using the data of large scale structure and CMB.

This paper is organized as follows. In Sec. II we consider a class of scalar-tensor theories with constant coupling Q . In Sec. III we study the background cosmological dynamics and consider the case of constant as well as varying λ (the slope of the potential in the physical frame). In this section we also introduce a family of potentials which are natural generalizations of a viable family of models in $f(R)$ gravity. In Sec. IV we discuss the LGC under the chameleon mechanism and place experimental bounds on parameters of viable scalar-tensor models using solar-system and equivalence principle constraints. In Sec. V we study the evolution of the equation of state of DE and show that the divergence of w_{DE} previously found in $f(R)$ theories is also present in the class of scalar-tensor models considered here which are compatible with LGC. In Sec. VI we discuss the evolution of density perturbations and place constraints on the coupling and model parameters employing the predicted difference in the slopes of the power spectra between large scale structure and the CMB. Finally we conclude in Sec. VII. The stability analysis which is crucial to derive the background cosmological scenario is briefly summarized in Appendix A 1. In Appendix A 2 we also clarify the stability condition of a de-Sitter point that appears in the presence of the coupling Q for varying λ .

II. SCALAR-TENSOR THEORIES

We start with a class of scalar-tensor theories, which includes the pure $f(R)$ theories as well as the quintessence models as special cases, in the form

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} f(\varphi, R) - \frac{1}{2} \zeta(\varphi) (\nabla \varphi)^2 \right] + S_m(g_{\mu\nu}, \Psi_m). \quad (1)$$

Here, f is a general differentiable function of the scalar field φ and the Ricci scalar R , ζ is a differentiable function of φ and S_m is a matter Lagrangian that depends on the metric $g_{\mu\nu}$ and matter fields Ψ_m . We also choose units such that $\kappa^2 \equiv 8\pi G = 1$, and restore the gravitational constant G when it makes the discussion more transparent.

The action (1) can be transformed to the so called Einstein frame under the conformal transformation [36] :

$$\tilde{g}_{\mu\nu} = e^{2\Omega} g_{\mu\nu}, \quad (2)$$

where

$$\Omega = \frac{1}{2} \ln F, \quad F = \frac{\partial f}{\partial R}. \quad (3)$$

In the following we shall consider F to be positive in order to ensure that gravity is attractive.

We shall be considering theories of the type

$$f(\varphi, R) = F(\varphi)R - 2V(\varphi), \quad (4)$$

for which the conformal factor Ω depends upon φ only. Introducing a new scalar field ϕ by

$$\phi = \int \left[\sqrt{\frac{3}{2} \left(\frac{F, \varphi}{F} \right)^2 + \frac{\zeta}{F}} \right] d\varphi, \quad (5)$$

the action in the Einstein frame becomes [36]

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2} \tilde{R} - \frac{1}{2} (\tilde{\nabla} \phi)^2 - U(\phi) \right] + S_m(\tilde{g}_{\mu\nu} F^{-1}, \Psi_m), \quad (6)$$

where a tilde represents quantities in the Einstein frame and

$$U = \frac{V}{F^2}. \quad (7)$$

In $f(R)$ gravity theories without the field φ , the conformal factor Ω depends only on R . Introducing a new scalar field to be

$$\phi = \frac{\sqrt{6}}{2} \ln F, \quad (8)$$

the action in the Einstein frame is given by (6) with the potential [36]

$$U = \frac{RF - f}{2F^2}. \quad (9)$$

Hence, the $f(R)$ gravity can be cast in the form of scalar-tensor theories of the type (1) with (4), by identifying the potential in the Jordan frame to be $V = (RF - f)/2$.

In order to describe the strength of the coupling between dark energy and a non-relativistic matter, we introduce the following quantity

$$Q = -\frac{F, \phi}{2F}. \quad (10)$$

From Eq. (8) one has $F = e^{2\phi/\sqrt{6}}$ which shows that the $f(R)$ gravity corresponds to

$$Q = -1/\sqrt{6}. \quad (11)$$

In what follows we shall study a class of scalar-tensor theories where Q is treated as an arbitrary constant. This class includes a wider family of models, including $f(R)$ gravity, induced gravity and quintessence models. Using Eqs. (5) and (10) we have the following relations

$$F = e^{-2Q\phi}, \quad \zeta = (1 - 6Q^2)F \left(\frac{d\phi}{d\varphi} \right)^2. \quad (12)$$

Then action (1) in the Jordan frame together with (4) yields

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2}FR - \frac{1}{2}(1 - 6Q^2)F(\nabla\phi)^2 - V \right] + S_m(g_{\mu\nu}, \Psi_m). \quad (13)$$

In $f(R)$ gravity the kinetic term of the field ϕ vanishes with the potential given by $V = (RF - f)/2$. Note that in the limit, $Q \rightarrow 0$, the action (13) reduces to the one for a minimally coupled scalar field ϕ with a potential $V(\phi)$.

It is informative to compare (13) with the following action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2}\chi R - \frac{\omega_{\text{BD}}}{2\chi}(\nabla\chi)^2 - V \right] + S_m(g_{\mu\nu}, \Psi_m), \quad (14)$$

which corresponds to Brans-Dicke theory with a potential V . Setting $\chi = F = e^{-2Q\phi}$, one easily finds that two actions are equivalent if the parameter ω_{BD} is related with Q via the relation

$$3 + 2\omega_{\text{BD}} = \frac{1}{2Q^2}. \quad (15)$$

Under this condition, the theories given by (13) are equivalent to the Brans-Dicke theory with a potential V .

In the following sections we shall in turn consider the evolution of the background dynamics in homogeneous settings, the local gravity constraints and the matter density perturbations.

III. HOMOGENEOUS COSMOLOGY

In what follows we shall discuss cosmological dynamics for the action (13) in the flat Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$, where t is cosmic time and $a(t)$ is the scale factor. As a source of the matter action S_m , we consider a non-relativistic fluid with energy density ρ_m and a radiation with energy density ρ_{rad} . Then the evolution equations in the Jordan frame are given by

$$3FH^2 = \frac{1}{2}(1 - 6Q^2)F\dot{\phi}^2 + V - 3H\dot{F} + \rho_m + \rho_{\text{rad}}, \quad (16)$$

$$2F\dot{H} = -(1 - 6Q^2)F\dot{\phi}^2 - \ddot{F} + H\dot{F} - \rho_m - \frac{4}{3}\rho_{\text{rad}}, \quad (17)$$

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (18)$$

$$\dot{\rho}_{\text{rad}} + 4H\rho_{\text{rad}} = 0, \quad (19)$$

where $H \equiv \dot{a}/a$ and a dot represents a derivative with respect to t .

Taking the time-derivative of Eq. (16) and using Eq. (17), we obtain

$$(1 - 6Q^2)F \left(\ddot{\phi} + 3H\dot{\phi} + \frac{\dot{F}}{2F}\dot{\phi} \right) + V_{,\phi} + QFR = 0, \quad (20)$$

where the Ricci scalar is given by

$$R = 6(2H^2 + \dot{H}). \quad (21)$$

We regard the Jordan frame as a physical one, since the usual matter conservation holds in this frame [see Eq. (18)].

In order to study the cosmological dynamics, it is convenient to introduce the following dimensionless phase space variables

$$x_1 = \frac{\dot{\phi}}{\sqrt{6}H}, \quad x_2 = \frac{1}{H}\sqrt{\frac{V}{3F}}, \quad x_3 = \frac{1}{H}\sqrt{\frac{\rho_{\text{rad}}}{3F}}. \quad (22)$$

Then the constraint equation (16) yields

$$\Omega_m \equiv \frac{\rho_m}{3FH^2} = 1 - (1 - 6Q^2)x_1^2 - x_2^2 - 2\sqrt{6}Qx_1 - x_3^2. \quad (23)$$

We also define the following quantities

$$\Omega_{\text{rad}} \equiv x_3^2, \quad \Omega_{\text{DE}} \equiv (1 - 6Q^2)x_1^2 + x_2^2 + 2\sqrt{6}Qx_1. \quad (24)$$

Eq. (23) then yields the relation $\Omega_m + \Omega_{\text{rad}} + \Omega_{\text{DE}} = 1$.

From Eqs. (17) and (20) we obtain

$$\frac{\dot{H}}{H^2} = -\frac{1 - 6Q^2}{2} \left[3 + 3x_1^2 - 3x_2^2 + x_3^2 - 6Q^2x_1^2 + 2\sqrt{6}Qx_1 \right] + 3Q(\lambda x_2^2 - 4Q), \quad (25)$$

$$\frac{\ddot{\phi}}{H^2} = 3(\lambda x_2^2 - \sqrt{6}x_1) + 3Q \left[(5 - 6Q^2)x_1^2 + 2\sqrt{6}Qx_1 - 3x_2^2 + x_3^2 - 1 \right]. \quad (26)$$

Using these relations, we obtain the following autonomous equations:

$$\frac{dx_1}{dN} = \frac{\sqrt{6}}{2}(\lambda x_2^2 - \sqrt{6}x_1) + \frac{\sqrt{6}Q}{2} \left[(5 - 6Q^2)x_1^2 + 2\sqrt{6}Qx_1 - 3x_2^2 + x_3^2 - 1 \right] - x_1 \frac{\dot{H}}{H^2}, \quad (27)$$

$$\frac{dx_2}{dN} = \frac{\sqrt{6}}{2}(2Q - \lambda)x_1x_2 - x_2 \frac{\dot{H}}{H^2}, \quad (28)$$

$$\frac{dx_3}{dN} = \sqrt{6}Qx_1x_3 - 2x_3 - x_3 \frac{\dot{H}}{H^2}, \quad (29)$$

where $N \equiv \ln(a)$ is the number of e-foldings and λ is defined by

$$\lambda \equiv -\frac{V_{,\phi}}{V}. \quad (30)$$

The exponential potential $V(\phi) = V_0 e^{-\lambda\phi}$ gives a constant value of λ . Generally, however, λ is dependent on ϕ , where the field ϕ is a function of x_1 , x_2 and x_3 through the definition of x_2 and Eq. (25). Hence Eqs. (27)-(29) are closed. The effective equation of state is given by

$$\begin{aligned} w_{\text{eff}} &\equiv -1 - \frac{2}{3} \frac{\dot{H}}{H^2} \\ &= -1 + \frac{1 - 6Q^2}{3} (3 + 3x_1^2 - 3x_2^2 + x_3^2 - 6Q^2x_1^2 + 2\sqrt{6}Qx_1) - 2Q(\lambda x_2^2 - 4Q). \end{aligned} \quad (31)$$

In what follows we shall first discuss the case of constant λ and then proceed to consider the varying λ case.

A. Constant λ

If λ is a constant, one can derive the fixed points of the system by setting the r.h.s. of Eqs. (27)-(29) to be zero. In the absence of radiation ($x_3 = 0$), we obtain the following fixed points:

- (a) ϕ matter-dominated era (ϕMDE [27])

$$(x_1, x_2) = \left(\frac{\sqrt{6}Q}{3(2Q^2 - 1)}, 0 \right), \quad \Omega_m = \frac{3 - 2Q^2}{3(1 - 2Q^2)^2}, \quad w_{\text{eff}} = \frac{4Q^2}{3(1 - 2Q^2)}. \quad (32)$$

- (b1) Kinetic point 1

$$(x_1, x_2) = \left(\frac{1}{\sqrt{6}Q + 1}, 0 \right), \quad \Omega_m = 0, \quad w_{\text{eff}} = \frac{3 - \sqrt{6}Q}{3(1 + \sqrt{6}Q)}. \quad (33)$$

- (b2) Kinetic point 2

$$(x_1, x_2) = \left(\frac{1}{\sqrt{6}Q - 1}, 0 \right), \quad \Omega_m = 0, \quad w_{\text{eff}} = \frac{3 + \sqrt{6}Q}{3(1 - \sqrt{6}Q)}. \quad (34)$$

- (c) Scalar-field dominated point

$$(x_1, x_2) = \left(\frac{\sqrt{6}(4Q - \lambda)}{6(4Q^2 - Q\lambda - 1)}, \left[\frac{6 - \lambda^2 + 8Q\lambda - 16Q^2}{6(4Q^2 - Q\lambda - 1)^2} \right]^{1/2} \right), \quad \Omega_m = 0, \quad w_{\text{eff}} = -\frac{20Q^2 - 9Q\lambda - 3 + \lambda^2}{3(4Q^2 - Q\lambda - 1)}. \quad (35)$$

- (d) Scaling solution

$$(x_1, x_2) = \left(\frac{\sqrt{6}}{2\lambda}, \left[\frac{3 + 2Q\lambda - 6Q^2}{2\lambda^2} \right]^{1/2} \right), \quad \Omega_m = 1 - \frac{3 - 12Q^2 + 7Q\lambda}{\lambda^2}, \quad w_{\text{eff}} = -\frac{2Q}{\lambda}. \quad (36)$$

- (e) de-Sitter point (present for $\lambda = 4Q$)

$$(x_1, x_2) = (0, 1), \quad \Omega_m = 0, \quad w_{\text{eff}} = -1. \quad (37)$$

Note that, when $x_3 \neq 0$ we have a radiation fixed point $(x_1, x_2, x_3) = (0, 0, 1)$.

One can easily confirm that the de-Sitter point exists for $\lambda = 4Q$, by setting $\dot{\phi} = 0$ in Eqs. (16), (17) and (20). This de-Sitter solution appears in the presence of the coupling Q . Note that this is the special case of the scalar-field dominated point (c).

Now given a value for λ , and using the stability conditions for the above fixed points given in the Appendix A, the cosmological dynamics can be specified. We shall briefly discuss the cases $Q = 0$ and $Q \neq 0$ in turn.

1. $Q = 0$

When $Q = 0$ (i.e., $F = 1$, which corresponds to a standard minimally coupled scalar field), the eigenvalues μ_1 and μ_2 of the Jacobian matrix for perturbations about the fixed points reduce to those derived in Ref. [37] with $\gamma = 1$ (see Ref. [38] for earlier works). In this case the matter-dominated era corresponds to either the point (a) or (d). The point (a) is a saddle node because $\mu_1 = -3/2$ and $\mu_2 = 3/2$. The point (d) is stable for $\lambda^2 > 3$, in which case $\Omega_m < 1$. The late-time accelerated expansion ($w_{\text{eff}} < -1/3$) can be realized by using the point (c), whose condition is given by $\lambda^2 < 2$. Under this condition the point (c) is a stable node. Hence, if $\lambda^2 < 2$, the saddle matter solution (a) is followed by the stable accelerated solution (c) [note that in this case $\Omega_m < 0$ for the point (d)]. The scaling solution (d) can have a matter era for $\lambda^2 \gg 1$, but in this case the epoch following the matter era is not of an accelerated nature.

2. $Q \neq 0$

We next consider the case of non-zero values of Q . Here we do not consider the special case of $\lambda = 4Q$. If the point (a) is responsible for the matter-dominated epoch, we require the condition $Q^2 \ll 1$. We then have $\Omega_m \simeq 1 + 10Q^2/3 > 1$ and $w_{\text{eff}} \simeq 4Q^2/3$, for the ϕ MDE. When $Q^2 \ll 1$ the scalar-field dominated point (c) yields an accelerated expansion provided that $-\sqrt{2} + 4Q < \lambda < \sqrt{2} + 4Q$ ¹. Under these conditions the ϕ MDE point is followed by the late-time acceleration. It is worth noting that in the case of $f(R)$ gravity ($Q = -1/\sqrt{6}$) the ϕ MDE point corresponds to, $\Omega_m = 2$ and $w_{\text{eff}} = 1/3$. In this case the universe in the matter era prior to late-time acceleration evolves as $a \propto t^{1/2}$, which is different from the evolution in the standard matter dominated epoch [9].

We note that the scaling solution (d) can give rise to the equation of state, $w_{\text{eff}} \simeq 0$ for $|Q| \ll |\lambda|$. In this case, however, the condition $w_{\text{eff}} < -1/3$ for the point (c) gives $\lambda^2 \lesssim 2$. Then the energy fraction of the pressureless matter for the point (d) does not satisfy the condition $\Omega_m \simeq 1$. In summary the viable cosmological trajectory corresponds to the sequence from the ϕ MDE to the scalar-field dominated point (c) under the conditions, $Q^2 \ll 1$ and $-\sqrt{2} + 4Q < \lambda < \sqrt{2} + 4Q$.

¹ Note that under the condition $Q^2 \ll 1$ and in the case where the dynamics is in the accelerated epoch, the condition $|Q\lambda| < 1$ is also satisfied.

B. Varying λ

When the time-scale of the variation of λ is smaller than that of the cosmic expansion, the fixed points derived above in the case of constant λ can be regarded as the “instantaneous” fixed points [39]. We shall briefly consider the cases of $Q = 0$ and $Q \neq 0$ in turn.

1. $Q = 0$

We begin with a brief discussion of the $Q = 0$ case. If the condition $\lambda^2 < 2$ is satisfied throughout the cosmic evolution, the cosmological trajectory is similar to the constant λ case discussed above except for the fact that the fixed points are regarded as the “instantaneous” ones. In this case the saddle matter solution (a) is followed by the accelerated point (c).

When $\lambda^2 \gg 1$ the scaling solution (d) is stable with $\Omega_m \simeq 1$. Hence the cosmological trajectory during the matter era chooses the scaling solution (d) rather than the saddle point (a). If $|\lambda|$ decreases at late-times, such that it satisfies the acceleration condition $\lambda^2 < 2$, the trajectory stops following the solution represented by the matter point (d) to follow the scalar-field dominated point (c)². A representative model of this type is provided by the double exponential potential, $V(\phi) = V_0(e^{-\lambda_1\phi} + e^{-\lambda_2\phi})$, with, $\lambda_1^2 \gg 1$ and $\lambda_2^2 < 2$ [40]. The assisted quintessence models in Ref. [41] also lead to a similar cosmological evolution.

2. $Q \neq 0$

We shall now proceed to consider the case of non-zero Q . If $|\lambda|$ is initially much larger than unity and decreases with time, it happens that the solutions finally approach the de-Sitter solution (e) with $\lambda = 4Q$. As we shall show in the Appendix A 2, the de-Sitter point (e) is in fact stable even for the variable λ case, if the potential satisfies the condition $Q(d\lambda/dF) > 0$ or $d\lambda/d\phi < 0$ at the de-Sitter point.

In the context of $f(R)$ gravity, it has been shown that the model,

$$f(R) = R - \mu R_c [1 - (R/R_c)^{-2n}] \quad (\mu > 0, R_c > 0, n > 0), \quad (38)$$

is a good example which can be consistent with cosmological and local gravity constraints [18]. Note that the models proposed by Hu & Sawicki [16] and Starobinsky [15] reduce to this form of $f(R)$ in the high-curvature region ($R \gg R_c$). In this model the field ϕ is related to the Ricci scalar R via the relation $e^{2\phi/\sqrt{6}} = 1 - 2n\mu(R/R_c)^{-(2n+1)}$. Hence, the potential $V = (FR - f)/2$ can be expressed in terms of the field ϕ as

$$V(\phi) = \frac{\mu R_c}{2} \left[1 - \frac{2n+1}{(2n\mu)^{2n/(2n+1)}} \left(1 - e^{2\phi/\sqrt{6}} \right)^{2n/(2n+1)} \right]. \quad (39)$$

The parameter λ is then given by

$$\lambda = -\frac{4n}{\sqrt{6}(2n\mu)^{2n/(2n+1)}} e^{2\phi/\sqrt{6}} \left[1 - \frac{2n+1}{(2n\mu)^{2n/(2n+1)}} \left(1 - e^{2\phi/\sqrt{6}} \right) \right]^{-2n/(2n+1)} \left(1 - e^{2\phi/\sqrt{6}} \right)^{-1/(2n+1)}. \quad (40)$$

In the deep matter-dominated epoch in which the condition $R/R_c \gg 1$ is satisfied, the field ϕ is very close to zero. For n and μ of the order of unity, $|\lambda|$ is much larger than unity during this stage. Hence the matter era is realized by the instantaneous fixed point (d). As R/R_c gets smaller, $|\lambda|$ decreases to the order of unity. If the solutions reach the point $\lambda = 4Q = -4/\sqrt{6}$ and satisfy the stability condition $d\lambda/dF < 0$ the final attractor corresponds to the de-Sitter fixed point (e).

For the theories with general couplings Q , let us consider the following scalar-field potential

$$V(\phi) = V_0 [1 - C(1 - e^{-2Q\phi})^p] \quad (V_0 > 0, C > 0, 0 < p < 1), \quad (41)$$

² Note that the de-Sitter solution (e) exists only for $\lambda = 0$, i.e., for the case of cosmological constant ($V = \text{const.}$).

as a natural generalization of Eq. (39). The slope of the potential is given by

$$\lambda = \frac{2CpQe^{-2Q\phi}(1 - e^{-2Q\phi})^{p-1}}{1 - C(1 - e^{-2Q\phi})^p}. \quad (42)$$

When $Q > 0$, the potential energy decreases from V_0 as ϕ increases from 0. On the other hand, if $Q < 0$, the potential energy decreases from V_0 as ϕ decreases from 0. In both cases we have $V(\phi) \rightarrow V_0(1 - C)$ in the limits $\phi \rightarrow \infty$ (for $Q > 0$) and $\phi \rightarrow -\infty$ (for $Q < 0$).

In the model (41) the field is stuck around the value $\phi = 0$ during the deep radiation and matter epochs. In these epochs one has $R \simeq \rho_m/F$ from Eqs. (16), (17) and (21) by noting that V_0 is negligibly small compared to ρ_m or ρ_{rad} . Using Eq. (20), we obtain the relation $V_{,\phi} + Q\rho_m \simeq 0$. Hence, in the high-curvature region the field ϕ evolves along the instantaneous minimum given by

$$\phi_m \simeq \frac{1}{2Q} \left(\frac{2V_0pC}{\rho_m} \right)^{\frac{1}{1-p}}. \quad (43)$$

We stress here that a range of minima appears depending upon the large energy density ρ_m of the non-relativistic matter. As long as the condition $\rho_m \gg V_0pC$ is satisfied, we have $|\phi_m| \ll 1$ from Eq. (43).

Since from Eq. (42) $|\lambda| \gg 1$ for field values around $\phi = 0$, the instantaneous fixed point (d) can represent the matter-dominated epoch provided that $|Q| \ll |\lambda|$. The deviation from Einstein gravity manifests itself when the field begins to evolve towards the end of the matter era. The variable $F = e^{-2Q\phi}$ decreases in time irrespective of the sign of the coupling strength and hence $0 < F < 1$. This decrease of F is crucial to the divergent behavior of the equation of state of DE, as we will see in Sec. V.

The de-Sitter solution corresponds to $\lambda = 4Q$, i.e.,

$$C = \frac{2}{(1 - F_1)^{p-1} [2 + (p - 2)F_1]}, \quad (44)$$

where F_1 is the value of F at the point (e). Provided that the solution of this equation exists in the region $0 < F_1 < 1$, for given values of C and p , the de-Sitter point exists. From Eq. (42) we obtain

$$\frac{d\lambda}{d\phi} = -\frac{4CpQ^2F(1 - F)^{p-2}[1 - pF - C(1 - F)^p]}{[1 - C(1 - F)^p]^2}. \quad (45)$$

When $0 < C < 1$, one can easily show that the function $g(F) \equiv 1 - pF - C(1 - F)^p$ is positive in the region $0 < F < 1$ giving $d\lambda/d\phi < 0$. Hence, the conditions for a stable de-Sitter point is automatically satisfied. In this case the solutions approach the de-Sitter attractor after the end of the matter era.

When $C > 1$, the function $g(F)$ becomes negative for values of F that are smaller than the critical value $F_c (< 1)$. The de-Sitter point (e) is stable under the condition $1 - pF_1 > C(1 - F_1)^p$. Using Eq. (44) we find that this stability condition translates to

$$F_1 > \frac{1}{2 - p}. \quad (46)$$

If this condition is violated, the solutions choose another stable fixed point as an attractor. In $f(R)$ gravity, for example, the solutions can reach the stable accelerated point (d) characterized by $m = -r - 1$ and $(\sqrt{3} - 1)/2 < m < 1$ [11], where $m \equiv Rf_{,RR}/f_{,R}$ and $r \equiv -Rf_{,R}/f$.

In summary, when $0 < C < 1$, the matter point (d) can be followed by the stable de-Sitter solution (e) for the model (41). In Fig. 1 we plot the evolution of Ω_{DE} , Ω_m , Ω_{rad} and w_{eff} for $Q = 0.01$, $p = 0.2$ and $C = 0.7$. Beginning from the epoch of matter-radiation equality, the solutions first dwell around the matter point (d) with $w_{\text{eff}} \simeq 0$ and finally approach the de-Sitter attractor (e) with $w_{\text{eff}} \simeq -1$. We have also numerically confirmed that λ is initially much larger than unity and eventually approaches the value $\lambda = 4Q$.

IV. LOCAL GRAVITY CONSTRAINTS

In this section we shall study the local gravity constraints (LGC) for the scalar-tensor theories given by the action (13). In the absence of the potential $V(\phi)$ the Brans-Dicke parameter ω_{BD} is constrained to be $\omega_{\text{BD}} > 4.0 \times 10^4$ from solar-system experiments [35]. Note that this bound also applies to the case of a nearly massless field with the

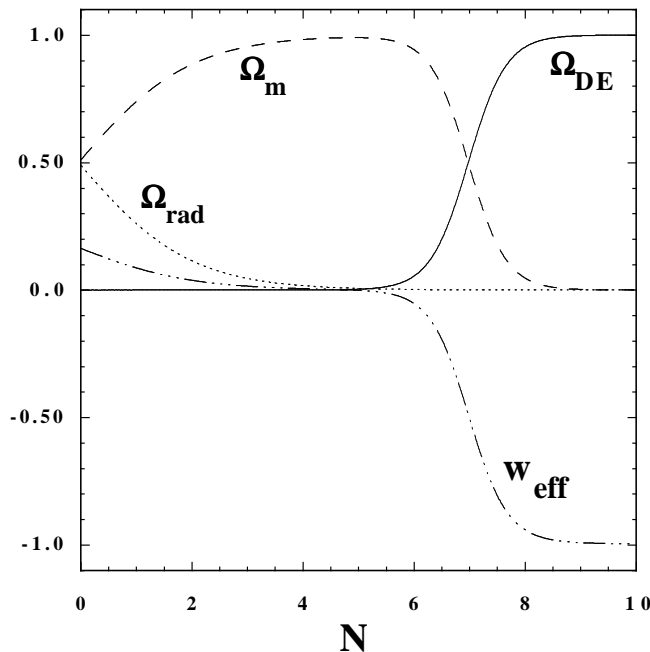


FIG. 1: The evolution of Ω_{DE} , Ω_m , Ω_{rad} and w_{eff} for the model (41) with parameters $Q = 0.01$, $p = 0.2$ and $C = 0.7$ and initial conditions $x_1 = 0$, $x_2 = 2.27 \times 10^{-7}$, $x_3 = 0.7$ and $x_4 - 1 = -5.0 \times 10^{-13}$.

potential $V(\phi)$ in which the Yukawa correction e^{-Mr} is close to unity (where M is the scalar field mass and r is an interaction length). Using the bound $\omega_{\text{BD}} > 4.0 \times 10^4$ in Eq. (15), we find

$$|Q| < 2.5 \times 10^{-3} \quad (\text{for the massless case}). \quad (47)$$

This is a strong constraint under which the cosmological evolution for such theories is difficult to be distinguished from the $Q = 0$ case.

Let us then consider the case in which the mass M of the field ϕ is sufficiently heavy so that the interaction range of the field ($\sim 1/M$) becomes short so as to satisfy LGC. In the context of $f(R)$ gravity ($Q = -1/\sqrt{6}$) it was in fact shown that the LGC can be satisfied by constructing models in which M is large enough in a high-dense region where local gravity experiments are carried out [15, 16, 17, 18, 19]. In these models the mass M tends to become lighter with the decrease of the Ricci scalar R towards the present epoch. In what follows, we shall construct viable models, based on scalar-tensor theories whose couplings Q are of order unity, which are consistent with LGC.

A. Chameleon mechanisms

In Refs. [12, 16, 24, 25] it was explicitly shown that in $f(R)$ gravity a spherically symmetric body forms a thin-shell inside the body through a chameleon mechanism [22, 23]. Generally this happens in a non-linear regime where the mass M of a scalar-field degree of freedom is heavy so that the usual linear analysis based on the inequality, $|\delta R| \ll |R^{(0)}|$, is invalid (where δR is a perturbation about a background value $R^{(0)}$). In what follows we shall briefly review the chameleon mechanism for the theory given in Eq. (13) and then place constraints on viable models consistent with LGC.

Let us consider the Einstein frame action (6). The variation of this action with respect to ϕ leads to the following equation of motion

$$\tilde{\square}\phi - U_{,\phi} = -Q\tilde{T}, \quad (48)$$

where, $\tilde{T} = e^{4Q\phi}T$ and $T = g^{\mu\nu}T_{\mu\nu}$, with $T_{\mu\nu}$ being energy momentum tensor of the matter in the Jordan frame. We take a spherically symmetric spacetime with a radius \tilde{r} from the center of symmetry. In this setup Eq. (48) becomes

$$\frac{d^2\phi}{d\tilde{r}^2} + \frac{2}{\tilde{r}} \frac{d\phi}{d\tilde{r}} = \frac{dU_{\text{eff}}}{d\phi}, \quad (49)$$

where

$$U_{\text{eff}}(\phi) = U(\phi) + e^{Q\phi} \rho^*. \quad (50)$$

Here, ρ^* is related with the energy density $\rho \equiv -T$ in the Jordan frame via the relation $\rho^* = e^{3Q\phi} \rho$, which is conserved in the Einstein frame [23] (i.e., $\rho^* \tilde{r}^3 = \text{constant}$).

We consider a configuration in which the spherically symmetric body has a constant density $\rho^* = \rho_A^*$ inside the body ($\tilde{r} < \tilde{r}_c$) and that the energy density outside the body ($\tilde{r} > \tilde{r}_c$) is given by $\rho^* = \rho_B^*$ ($\ll \rho_A^*$). Then the mass of this body is given by $M_c = (4\pi/3)\tilde{r}_c^3 \rho_A^* = (4\pi/3)r_c^3 \rho_A$. Let us denote the field value at the minimum of the effective potential $U_{\text{eff}}(\phi)$ corresponding to the density ρ_A^* (ρ_B^*) by ϕ_A (ϕ_B). That is, they are given by

$$U_{,\phi}(\phi_A) + Qe^{Q\phi_A} \rho_A^* = 0, \quad \text{and} \quad U_{,\phi}(\phi_B) + Qe^{Q\phi_B} \rho_B^* = 0, \quad (51)$$

respectively. Under the condition $\rho_A^* \gg \rho_B^*$ the mass squared $m_A^2 \equiv U''_{\text{eff}}(\phi_A)$ is much larger than $m_B^2 \equiv U''_{\text{eff}}(\phi_B)$.

In solving for a static spherically symmetric field configuration, we impose the boundary conditions $d\phi(\tilde{r} = 0)/d\tilde{r} = 0$ and $\phi(\tilde{r} \rightarrow \infty) = \phi_B$, so that the field ϕ is non-singular at $\tilde{r} = 0$ and that the force on a test body vanishes at a sufficiently large distance. Then ϕ starts to roll down the potential where the term $dU_{\text{eff}}/d\phi$ becomes important.

If the field value at the center $\phi(\tilde{r} = 0)$ is close enough to the equilibrium value ϕ_A with $|\phi(\tilde{r} = 0) - \phi_A| \ll |\phi_A|$, the thin-shell solution is realized [23]. In this case the field does not move away from $\phi(\tilde{r} = 0)$ practically up to a radius \tilde{r}_1 which satisfies $\Delta\tilde{r}_c/\tilde{r}_c \equiv (\tilde{r}_c - \tilde{r}_1)/\tilde{r}_c \ll 1$. At $\tilde{r} = \tilde{r}_1$, the field starts to roll down the potential and we find, $|U_{,\phi}(\phi)| \ll |Qe^{Q\phi} \rho_A^*|$ for $\tilde{r}_1 < \tilde{r} < \tilde{r}_c$. Under the condition $|Q\phi_A| \ll 1$ (as we will confirm later) the r.h.s. of Eq. (49) is approximately given by $dU_{\text{eff}}/d\phi \simeq Q\rho_A^*$. Outside the body ($\tilde{r} > \tilde{r}_c$) the gradient energies on the l.h.s. of Eq. (49) become important because the energy density drops down from ρ_A^* to ρ_B^* . Taking into account the mass term m_B of the effective potential U_{eff} , one has $dU_{\text{eff}}/d\phi = m_B^2(\phi - \phi_B)$ on the r.h.s. of Eq. (49).

We match the solutions of Eq. (49) at $\tilde{r} = \tilde{r}_c$ with the boundary conditions $\phi = \phi_A$, $d\phi/d\tilde{r} = 0$ at $\tilde{r} = \tilde{r}_1$ and $\phi(\tilde{r} \rightarrow \infty) = \phi_B$. Then the following solution is obtained in the region $\tilde{r} > \tilde{r}_c$ [23, 25].

$$\phi(\tilde{r}) \simeq -\frac{QM_c}{4\pi} \left[1 - \left(\frac{\tilde{r}_1}{\tilde{r}_c} \right)^3 \right] \frac{e^{-m_B(\tilde{r}-\tilde{r}_c)}}{\tilde{r}} + \phi_B, \quad (52)$$

where

$$\left(\frac{\tilde{r}_1}{\tilde{r}_c} \right)^2 \simeq 1 - \frac{\phi_B - \phi_A}{3Q\Phi_c}, \quad \Phi_c \equiv \frac{M_c}{8\pi\tilde{r}_c} = \frac{GM_c}{\tilde{r}_c}. \quad (53)$$

In deriving the relation (52), we assumed the condition $m_B\tilde{r}_c \ll 1$. Since we are in the thin-shell regime, we obtain the following relation from Eq. (53).

$$\frac{\Delta\tilde{r}_c}{\tilde{r}_c} \simeq \frac{\phi_B - \phi_A}{6Q\Phi_c}. \quad (54)$$

Then the solution outside the body ($\tilde{r} > \tilde{r}_c$) is given by

$$\phi(\tilde{r}) \simeq -\frac{Q_{\text{eff}}}{4\pi} \frac{M_c e^{-m_B(\tilde{r}-\tilde{r}_c)}}{\tilde{r}} + \phi_B, \quad \text{where} \quad Q_{\text{eff}} \equiv 3Q \frac{\Delta\tilde{r}_c}{\tilde{r}_c}. \quad (55)$$

Thus, when the body has a thin-shell, the effective coupling $|Q_{\text{eff}}|$ in the thin-shell becomes much smaller than unity, even if $|Q|$ itself is of the order of 1.

If the field value at $\tilde{r} = 0$ is not close to ϕ_A (i.e., $|\phi(\tilde{r} = 0) - \phi_A| \gtrsim |\phi_A|$), the field rapidly rolls down the potential at $\tilde{r}_1 \simeq 0$. Setting $\tilde{r}_1 = 0$ in Eq. (52), we obtain the solution (55) with Q_{eff} replaced by Q . This is the thick-shell regime in which the effective coupling is not small as to satisfy the LGC.

The presence of the fifth force interaction mediated by the field ϕ leads to a modification to the spherically symmetric metric. Under the weak field approximation, the spherically symmetric metric in the Jordan frame is given by [12, 25]

$$ds^2 = - \left(1 - \frac{2G_{\text{eff}}M_c}{r} \right) dt^2 + \left(1 + \frac{2\gamma G_{\text{eff}}M_c}{r} \right) dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (56)$$

where the effective gravitational “constant” G_{eff} and the post-Newtonian parameter γ are given by

$$G_{\text{eff}} \simeq G \left[1 - \frac{\sqrt{6}}{3} Q_{\text{eff}} e^{-m_B(r-r_c)} \right], \quad \gamma \simeq \frac{1 + (\sqrt{6}Q_{\text{eff}}/3)(1 + m_B r) e^{-m_B(r-r_c)}}{1 - (\sqrt{6}Q_{\text{eff}}/3) e^{-m_B(r-r_c)}}. \quad (57)$$

Note that we have used the approximation $\tilde{r} \simeq r$ that is valid in the region $|Q\phi| \ll 1$.

Provided that the condition $m_B r \ll 1$ holds in an environment in which local gravity experiments are carried out, we have $\gamma \simeq (1 + \sqrt{6}Q_{\text{eff}}/3)/(1 - \sqrt{6}Q_{\text{eff}}/3)$. Hence, if $|Q_{\text{eff}}|$ is much smaller than unity through the chameleon mechanism, it is possible to satisfy the following severest solar system constraint that comes from a time-delay effect of the Cassini tracking [42]:

$$|\gamma - 1| < 2.3 \times 10^{-5}. \quad (58)$$

Using the thin-shell parameter, this bound translates into

$$\frac{\Delta r_c}{r_c} < \frac{4.7 \times 10^{-6}}{|Q|}. \quad (59)$$

If the body does not have a thin-shell for $|Q|$ of the order of unity, the condition (58) is not satisfied. In $f(R)$ gravity, for example, we have $\gamma = 1/2$ for $Q_{\text{eff}} = Q = -1/\sqrt{6}$.

B. Solar system constraints

In $f(R)$ gravity, the models (38) can satisfy LGC because the mass M of the field potential (39) is sufficiently heavy in the high-density region whose Ricci scalar R is much larger than R_c . Since the field mass M_ϕ inside the body is much heavier than that outside the body, most of the volume element within the core does not contribute to the field profile at $r > r_c$ except for the thin-shell regime around the surface of the body (note that this contribution is proportional to $e^{-M\ell}$, where ℓ is a distance from the volume element to a point outside the body). In the case of general couplings Q , the models presented in Eq. (41) can be compatible with LGC. Under the condition $|Q\phi| \ll 1$, one has $U_{,\phi} \simeq -2V_0 Q p C (2Q\phi)^{p-1}$ for the potential $U = V/F^2$ in the Einstein frame. Then from Eq. (51) we obtain the field values at the potential minima inside/outside the body:

$$\phi_A \simeq \frac{1}{2Q} \left(\frac{2V_0 p C}{\rho_A} \right)^{\frac{1}{1-p}}, \quad \phi_B \simeq \frac{1}{2Q} \left(\frac{2V_0 p C}{\rho_B} \right)^{\frac{1}{1-p}}, \quad (60)$$

which satisfy $|\phi_A| \ll |\phi_B|$. Note that these are analogous to the field value ϕ_m derived in Eq. (43) in the cosmological setting. In order to realize the accelerated expansion at the present epoch, V_0 needs to be roughly the same order as the square of the present Hubble parameter H_0 , so we have $V_0 \sim H_0^2 \sim \rho_0$, where $\rho_0 \simeq 10^{-29} \text{ g/cm}^3$ is the present cosmological density. Note that the baryonic/dark matter density in our galaxy corresponds to $\rho_B \simeq 10^{-24} \text{ g/cm}^3$. This then shows that the conditions, $|Q\phi_A| \ll 1$ and $|Q\phi_B| \ll 1$ are in fact satisfied provided that C is not much larger than unity.

The field mass squared $M_\phi^2 \equiv d^2 U / d\phi^2$ at $\phi = \phi_A$ is approximately given by

$$M_\phi^2(\phi_A) \simeq \frac{1-p}{(2^p p C)^{1/(1-p)}} Q^2 \left(\frac{\rho_A}{V_0} \right)^{\frac{2-p}{1-p}} V_0. \quad (61)$$

This means that $M_\phi(\phi_A)$ can be much larger than H_0 due to the condition $\rho_A \gg V_0$. Therefore, while the mass M_ϕ is not different from the order of H_0 on cosmological scales, it increases in the regions with a higher energy density.

Let us place constraints on model parameters by using the solar system bound (59). In so doing, we shall consider the case where the solutions finally approach the de-Sitter point (e). Since we have $\Delta r_c / r_c \simeq \phi_B / (6Q\Phi_c)$ with ϕ_B given in Eq. (60), the bound (59) translates into

$$(2V_0 p C / \rho_B)^{1/(1-p)} < 1.2 \times 10^{-10} |Q|, \quad (62)$$

where we have used the value $\Phi_c = 2.12 \times 10^{-6}$ for the Sun. At the de-Sitter point (e), one has $3F_1 H_1^2 = V_0 [1 - C(1 - F_1)^p]$ with C given in Eq. (44). Hence, we obtain the following relation

$$V_0 = 3H_1^2 \frac{2 + (p-2)F_1}{p}. \quad (63)$$

Substituting this in Eq. (62) we find

$$\left(\frac{R_1}{\rho_B} \right)^{1/(1-p)} (1 - F_1) < 1.2 \times 10^{-10} |Q|, \quad (64)$$

where $R_1 = 12H_1^2$ is the Ricci scalar at the de-Sitter point. Since the term $(1 - F_1)$ is smaller than one half from the condition (46) we obtain the inequality $(R_1/\rho_B)^{1/(1-p)} < 2.4 \times 10^{-10}|Q|$. We assume that R_1 is of the order of the present cosmological density $\rho_0 = 10^{-29}$ g/cm³. Taking the baryonic/dark matter density to be $\rho_B = 10^{-24}$ g/cm³ outside the Sun we obtain the following bound

$$p > 1 - \frac{5}{9.6 - \log_{10}|Q|}. \quad (65)$$

For $|Q| = 10^{-2}$ and $|Q| = 10^{-1}$ this gives $p > 0.57$ and $p > 0.53$ respectively. The above bound corresponds to $p > 0.50$ for the case of $f(R)$ gravity, which translates into the condition $n > 0.5$ in Eq. (39). This agrees with the result found in Ref. [25].

C. Equivalence principle constraints

Let us proceed to the constraints from a possible violation of the equivalence principle (EP). Under the condition that the neighborhood of the Earth has a thin-shell, the tightest bound comes from solar system tests of the EP that makes use of the free-fall accelerations of the Moon (a_{Moon}) and the Earth (a_{\oplus}) toward the Sun [23, 25]. The bound on the differences between the two accelerations is [42]

$$2 \frac{|a_{\text{Moon}} - a_{\oplus}|}{a_{\text{Moon}} + a_{\oplus}} < 10^{-13}. \quad (66)$$

Since the acceleration induced by a fifth force with the field profile $\phi(r)$ and the effective coupling is given by $a_{\text{ffth}} = |Q_{\text{eff}}\phi(r)|$ we obtain [23]

$$a_{\oplus} = \frac{GM_{\odot}}{r^2} \left[1 + 3 \left(\frac{\Delta r_{\oplus}}{r_{\oplus}} \right)^2 \frac{\Phi_{\oplus}}{\Phi_{\odot}} \right], \quad a_{\text{Moon}} = \frac{GM_{\odot}}{r^2} \left[1 + 3 \left(\frac{\Delta r_{\oplus}}{r_{\oplus}} \right)^2 \frac{\Phi_{\oplus}^2}{\Phi_{\odot}\Phi_{\text{Moon}}} \right], \quad (67)$$

where $\Phi_{\odot} \simeq 2.1 \times 10^{-6}$, $\Phi_{\oplus} \simeq 7.0 \times 10^{-10}$ and $\Phi_{\text{Moon}} \simeq 3.1 \times 10^{-11}$, are the gravitational potentials of the Sun, the Earth and the Moon, respectively. Note that $\Delta r_{\oplus}/r_{\oplus}$ is the thin-shell parameter of the Earth. From the bound (66), this is constrained to be

$$\frac{\Delta r_{\oplus}}{r_{\oplus}} < \frac{8.8 \times 10^{-7}}{|Q|}. \quad (68)$$

Note also that the thin-shell condition for the neighborhood outside the Earth provides the same order of the upper bound for, $\Delta r_{\oplus}/r_{\oplus}$ [25].

Taking a similar procedure as in the case of the solar system constraints discussed above (using the value $R_1 = 10^{-29}$ g/cm³ and $\rho_B = 10^{-24}$ g/cm³), we obtain the following bound:

$$p > 1 - \frac{5}{13.8 - \log_{10}|Q|}. \quad (69)$$

This is tighter than the bound (65). When $|Q| = 10^{-2}$ and $|Q| = 10^{-1}$ we have $p > 0.68$ and $p > 0.66$, respectively. In the case of $f(R)$ gravity the above bound corresponds to $p > 0.65$ which translates to $n > 0.9$ for the potential (39).

In summary, the LGC can be satisfied under the condition (69) for the potential (41).

D. General properties for models consistent with LGC

In this subsection we shall consider the general properties of scalar-tensor theories consistent with LGC, without specifying the form of the field potential. In order to satisfy the LGC we require that $|\phi_B - \phi_A|$ is much smaller than $|Q\Phi_c|$ from Eq. (54). Since there is a gap between the energy densities inside and outside of the spherically symmetric body we have, $|\phi_B - \phi_A| \simeq |\phi_B|$, which implies $|\phi_B| \ll |Q\Phi_c|$. The gravitational potential Φ_c is very much smaller than unity in settings where local gravity experiments are carried out, hence this yields the constraint $|\phi_B| \ll 1$. Cosmologically this means that $|\phi|$ is much smaller than unity during matter/radiation epochs. When $|Q| \gg 1$ the condition $|\phi_B| \ll 1$ is not necessarily ensured, but those cases are excluded by the constraints from

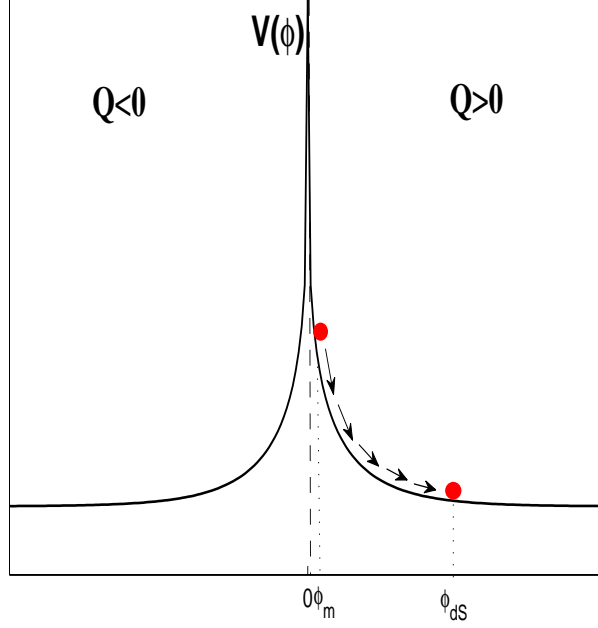


FIG. 2: This illustration describes a field potential $V(\phi)$ that is consistent with LGC. For a coupling Q that is positive (negative) the potential evolves in the region $\phi \geq 0$ ($\phi \leq 0$). In the figure ϕ_m represents the field value during the radiation/matter eras, which instantaneously changes in time. The field value ϕ_{dS} corresponds to the one at the de-Sitter point. Note that both ϕ_m and ϕ_{dS} are sustained by the presence of the coupling Q having potential minima characterized by the condition (70). In the early stages of the cosmological evolution, the mass M of the field ϕ is heavy for consistency with LGC. This mass gradually gets smaller as the system approaches the de-Sitter point.

density perturbations unless the model is very close to the Λ CDM model (as we shall see later). In the following we shall consider the theories with $|Q| \lesssim 1$.

In the region $|\phi| \ll 1$ (i.e., $F \simeq 1$), the derivative terms are negligible in Eq. (20) and the field stays at instantaneous minima given by

$$V_{,\phi} + QFR = 0, \quad (70)$$

in the late radiation-dominated and matter-dominated eras. The condition (70) translates into $\lambda/Q = \rho_m/V$ which means that $\lambda/Q \gg 1$ in the radiation and matter epochs. This is in fact consistent with the condition $|w_{\text{eff}}| = |2Q/\lambda| \ll 1$ for the existence of a viable matter point (d). If the de-Sitter point (e) is stable, the solutions finally approach the minimum given by (70), i.e., $\lambda/Q = 4$.

The sign of λ needs to be the same as that of Q in order to realize the above cosmological trajectory. When $Q > 0$, we require $\lambda = -V_{,\phi}/V > 0$, i.e., $V_{,\phi} < 0$, which means that the field ϕ evolves along the potential toward larger positive values from $\phi \simeq 0$. When $Q < 0$ the field evolves towards smaller negative values from $\phi \simeq 0$.

We illustrate such potentials in Fig. 2. Since the ratio λ/Q decreases from the radiation/matter epochs to the de-Sitter epoch, the derivative $d\lambda/d\phi$ is negative irrespective of the sign of Q . We recall that in this case the stability of the de-Sitter point (e) is also ensured. Since $d\lambda/d\phi = \lambda^2 - V_{,\phi\phi}/V$, the mass squared

$$M^2 \equiv V_{,\phi\phi}, \quad (71)$$

is required to be positive to satisfy the condition $d\lambda/d\phi < 0$. Moreover, the mass M needs to be heavy enough in order to satisfy the condition $M^2 > \lambda^2 V$ in radiation/matter epochs. The model (41) provides a representative example which satisfies all the requirements discussed above.

It is worth mentioning that for the models that satisfy LGC, the quantity $F = e^{-2Q\phi}$ in the matter/radiation eras is larger than its value at the de-Sitter point. It is this property which leads to an interesting observational signature for the DE equation of state, as we shall see in the next section.

V. THE EQUATION OF STATE OF DARK ENERGY

In scalar-tensor DE models, a meaningful definition of energy density and pressure of DE requires some care. In this section, following Ref. [28], we shall discuss the evolution of the equation of state of DE, which could provide comparisons with observations. In the absence of radiation, Eqs. (16) and (17) can be written as

$$3F_0 H^2 = \rho_{\text{DE}} + \rho_m, \quad (72)$$

$$-2F_0 \dot{H} = \rho_{\text{DE}} + p_{\text{DE}} + \rho_m, \quad (73)$$

where the subscript “0” represents present values and

$$\rho_{\text{DE}} \equiv \frac{1}{2}(1 - 6Q^2)F\dot{\phi}^2 + V - 3H\dot{F} - 3(F - F_0)H^2, \quad (74)$$

$$p_{\text{DE}} \equiv \frac{1}{2}(1 - 6Q^2)F\dot{\phi}^2 - V + \ddot{F} + 2H\dot{F} + (F - F_0)(3H^2 + 2\dot{H}), \quad (75)$$

which satisfy the usual conservation equation

$$\dot{\rho}_{\text{DE}} + 3H(\rho_{\text{DE}} + p_{\text{DE}}) = 0. \quad (76)$$

We define the equation of state of DE to be

$$w_{\text{DE}} \equiv \frac{p_{\text{DE}}}{\rho_{\text{DE}}} = \frac{w_{\text{eff}}}{1 - (F/F_0)\Omega_m}, \quad (77)$$

where Ω_m and w_{eff} are defined in Eqs. (23) and (31), respectively. Integrating Eq. (18), we obtain

$$\rho_m = 3F_0\Omega_m^{(0)}H_0^2(1+z)^3, \quad (78)$$

where $\Omega_m^{(0)}$ is the present energy fraction of the non-relativistic matter and $z \equiv a_0/a - 1$ is the redshift. On using Eqs. (72) and (73), we find

$$w_{\text{DE}} = -\frac{3r - (1+z)(dr/dz)}{3r - 3\Omega_m^{(0)}(1+z)^3}, \quad (79)$$

where $r = H^2(z)/H_0^2$. Note that this is the same equation as the one used in Einstein gravity [2]. By defining the energy density ρ_{DE} and the pressure p_{DE} as given in Eqs. (74) and (75), the resulting DE equation of state w_{DE} agrees with the usual expression which can be used to confront the models with SNIa observations.

From Eq. (77) we find that w_{DE} becomes singular at the point $\Omega_m = F_0/F$. This happens for models in which F increases from its present value F_0 as we go back in time. From Eq. (12) it is clear that F decreases in time for $Q\dot{\phi} > 0$. We note that even when the system crosses the point $\Omega_m = F_0/F$ physical quantities such as the Hubble parameter remain to be continuous.

The models (41) satisfy this condition regardless of the sign of Q , which means that the divergent behavior of w_{DE} indeed occurs. We recall that in the context of $f(R)$ gravity ($Q = -1/\sqrt{6}$) the models $f(R) = R - \mu^{2(n+1)}/R^n$ ($n > 0$) correspond to a scalar field potential that decreases toward larger ϕ , i.e., $\dot{\phi} > 0$ [6]. Hence, the divergence of w_{DE} does not occur in such models because of the decrease of F toward the past.

For the models that satisfy $|\lambda| \gg 1$ initially such that $|\lambda|$ decreases with time, the solutions are in the regime around the instantaneous fixed point (d) during the matter era and finally approach either the scalar-field dominated point (c) or the de-Sitter point (e). In Fig. 3 we plot the evolution of w_{DE} for the case $Q = 0.1$ and $C = 0.95$, with three different values of p . In these cases the final attractor corresponds to the de-Sitter point (e) satisfying the relation $\lambda = 0.4$. During the deep matter era the solutions evolve along the “instantaneous” fixed point (d) with Ω_m close to 1 (because $\lambda \gg 1$). After λ decreases to the order of unity, the solutions approach the de-Sitter solution (e) with $\Omega_m = 0$ and $w_{\text{DE}} = w_{\text{eff}} = -1$.

Figure 3 clearly shows that w_{DE} exhibits a divergence at a redshift z_c that depends on the values of p . When $p = 0.3$, for example, the divergence occurs around the redshift, $z_c = 3$. For compatibility with LGC we require $p > 0.53$, from solar system constraints, and $p > 0.66$ from EP constraints, as we showed in the previous section. In those cases the critical redshift z_c gets larger, which is out of the observational range of current SNIa observations. Nevertheless, the equation of state w_{DE} shows a peculiar evolution that changes from $w_{\text{DE}} < -1$ to $w_{\text{DE}} > -1$ at a redshift around $z_c = \mathcal{O}(1)$. This cosmological boundary crossing, similar to the divergence of w_{DE} , is attributed to

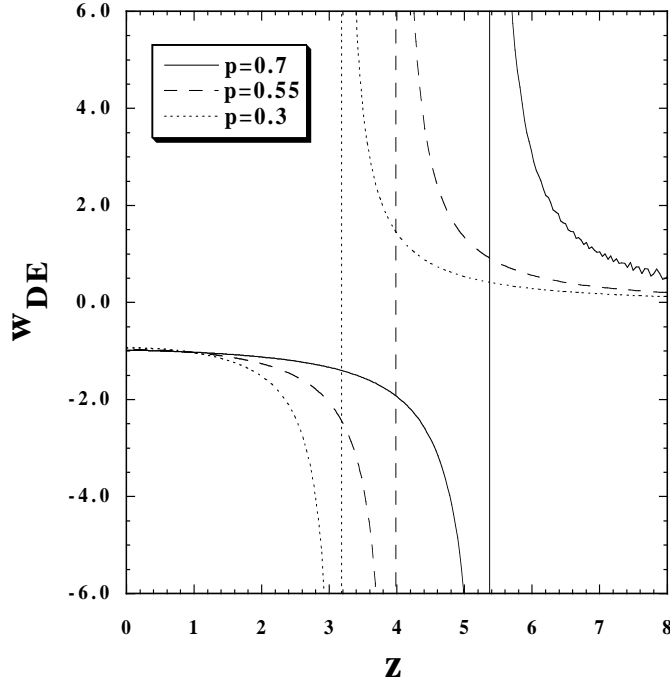


FIG. 3: Figure depicting the evolution of w_{DE} for $Q = 0.1$ and $C = 0.95$ with three different values of p (0.3, 0.55, 0.7). The redshift z_c at which the divergence of w_{DE} occurs decreases for smaller p .

the fact that F increases as we go back to the past. It is worth noting that this is a common feature among viable models that are consistent with LGC, as we have illustrated in the previous section.

Note that in the limit $Q \rightarrow 0$ the potential $V(\phi)$ approaches a constant value $V(\phi) \rightarrow V_0(1 - C)$. Hence, the models are hardly distinguishable from the Λ CDM model. In these cases the critical redshift z_c also goes to infinity. Thus, the effect of modified gravity is more apparent for larger $|Q|$ and smaller p . In $f(R)$ gravity, for example, the model given by Eq. (39) can give rise to the redshift z_c as close as a few [18] while satisfying the LGC ($p > 0.65$). These cases are particularly interesting to place tight bounds on model parameters from future high-precision observations.

VI. MATTER DENSITY PERTURBATIONS

In this section we discuss the evolution of matter density perturbations and the resulting spectra for scalar-tensor theories. Considering as our background spacetime the flat FLRW metric, the perturbed metric including the scalar metric perturbations Φ and Ψ in the longitudinal gauge is given by [43]

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)dx^i dx^j. \quad (80)$$

In the following we shall neglect radiation and consider only the pressureless matter. The components of the energy momentum tensor of the pressureless matter are given by

$$T_0^0 = -(\rho_m + \delta\rho_m), \quad T_i^0 = -\rho_m v_{m,i}, \quad (81)$$

where v_m is related to the velocity potential V through $v_m = -(V - b)$. In the Fourier space matter perturbations satisfy the following equations of motion [19, 34, 44]:

$$\Phi = \dot{v}, \quad (82)$$

$$(\delta\rho_m/\rho_m)' = 3\dot{\Psi} - \frac{k^2}{a^2}v, \quad (83)$$

where $v \equiv av_m$ is a covariant velocity perturbation and k is a comoving wavenumber. We shall introduce the following gauge-invariant density contrast:

$$\delta_m = \frac{\delta\rho_m}{\rho_m} + 3Hv. \quad (84)$$

The evolution equation for δ_m is then given by

$$\ddot{\delta}_m + 2H\dot{\delta}_m + \frac{k^2}{a^2}\Phi = 3\ddot{B} + 6H\dot{B}, \quad (85)$$

where $B = Hv + \Psi$.

In Fourier space the scalar metric perturbations in scalar-tensor theories satisfy the following equations [44]

$$\begin{aligned} \frac{k^2}{a^2}\Psi + 3H(H\Phi + \dot{\Psi}) = -\frac{1}{2F} \left[\omega\dot{\phi}\delta\dot{\phi} + \frac{1}{2}(\omega_{,\phi}\dot{\phi}^2 - F_{,\phi}R + 2V_{,\phi})\delta\phi - 3H\delta\dot{F} + \left(3\dot{H} + 3H^2 - \frac{k^2}{a^2}\right)\delta F \right. \\ \left. + (3H\dot{F} - \omega\dot{\phi}^2)\Phi + 3\dot{F}(H\Phi + \dot{\Psi}) + \delta\rho_m \right], \end{aligned} \quad (86)$$

$$H\Phi + \dot{\Psi} = \frac{1}{2F} \left(\omega\dot{\phi}\delta\phi + \delta\dot{F} - H\delta F - \dot{F}\Phi + \rho_mv \right), \quad (87)$$

$$\Psi - \Phi = \frac{\delta F}{F}, \quad (88)$$

$$\begin{aligned} \delta\ddot{\phi} + \left(3H + \frac{\omega_{,\phi}}{\omega}\dot{\phi}\right)\delta\dot{\phi} + \left[\frac{k^2}{a^2} + \left(\frac{\omega_{,\phi}}{\omega}\right)_{,\phi}\frac{\dot{\phi}^2}{2} + \left(\frac{2V_{,\phi} - F_{,\phi}R}{2\omega}\right)_{,\phi}\right]\delta\phi \\ = \dot{\phi}\dot{\Phi} + \left(2\ddot{\phi} + 3H\dot{\phi} + \frac{\omega_{,\phi}}{\omega}\dot{\phi}^2\right)\Phi + 3\dot{\phi}(H\Phi + \dot{\Psi}) + \frac{1}{2\omega}F_{,\phi}\delta R, \end{aligned} \quad (89)$$

where $\omega = (1 - 6Q^2)F$ and

$$\delta R = 2 \left[-3(H\Phi + \dot{\Psi}) - 12H(H\Phi + \dot{\Psi}) + \left(\frac{k^2}{a^2} - 3\dot{H}\right)\Phi - 2\frac{k^2}{a^2}\Psi \right]. \quad (90)$$

As long as the mass M defined in Eq. (71) is sufficiently heavy to satisfy the conditions $M^2 \gg R$ and $M^2 > \lambda^2 V$ (in order to ensure $d\lambda/d\phi < 0$), one can approximate $((2V_{,\phi} - F_{,\phi}R)/2\omega)_{,\phi} \simeq M^2/\omega$ in Eq. (89). While this quantity becomes negative for $Q^2 > 1/6$ this does not imply that the perturbation $\delta\phi$ exhibits a negative instability. In fact we shall illustrate below, that due to the perturbation δR on the r.h.s. of Eq. (89), the effective mass produced is positive.

Generally, the solution of Eq. (89) consists of the sum of the matter-induced mode $\delta\phi_{\text{ind}}$ sourced by the matter perturbation and the oscillating mode $\delta\phi_{\text{osc}}$ (scalarons [5]), i.e., $\delta\phi = \delta\phi_{\text{ind}} + \delta\phi_{\text{osc}}$. The oscillating mode corresponds to the solution of Eq. (89) without the matter perturbation.

In order to derive the approximate perturbation equations on sub-horizon scales, we use the approximation according to which the terms containing k^2/a^2 , $\delta\rho_m$, δR and M^2 dominate in Eqs. (86)-(89). This method was used in Refs. [2, 34, 45] in the nearly massless case ($M^2 \lesssim H^2$). In the context of $f(R)$ gravity this approximation was shown in Ref. [19] to be extremely accurate even in the massive case ($M^2 \gg H^2$) as long as the oscillating degrees of freedom do not dominate over the matter-induced mode.

In order to extract the peculiar features of the matter perturbations in scalar-tensor tensor theories, let us first concentrate on the matter induced mode. Under the above-mentioned approximation, we have $\delta R_{\text{ind}} \simeq -2(k^2/a^2)[\Psi + (F_{,\phi}/F)\delta\phi_{\text{ind}}]$ from Eqs. (88) and (90), where the subscript ‘‘ind’’ represents a matter induced mode. Then from Eq. (89) we find

$$\delta\phi_{\text{ind}} \simeq \frac{2QF}{(k^2/a^2)(1 - 2Q^2)F + M^2} \frac{k^2}{a^2} \Psi. \quad (91)$$

Using Eq. (86) and (88) we obtain

$$\frac{k^2}{a^2}\Psi \simeq -\frac{\delta\rho_m}{2F} \frac{(k^2/a^2)(1 - 2Q^2)F + M^2}{(k^2/a^2)F + M^2}, \quad \frac{k^2}{a^2}\Phi \simeq -\frac{\delta\rho_m}{2F} \frac{(k^2/a^2)(1 + 2Q^2)F + M^2}{(k^2/a^2)F + M^2}. \quad (92)$$

In the limit $M^2/F \gg k^2/a^2$ one has $(k^2/a^2)\Phi \simeq -\delta\rho_m/2F \simeq -4\pi G\delta\rho_m$, which recovers the standard Poisson equation. In the limit $M^2/F \ll k^2/a^2$ one has $(k^2/a^2)\Phi \simeq -(\delta\rho_m/2F)(1 + 2Q^2)$, where the effect of the coupling Q becomes important.

From Eq. (87) we find that v is of the order of $FH\Phi/\rho_m$. Using the fact that $(k^2/a^2)\Phi$ is of the order of $-(1/F)\delta\rho_m$ we can estimate that $|3Hv/(\delta\rho_m/\rho_m)| \sim (aH)^2/k^2 \ll 1$. Hence we have $\delta_m \simeq \delta\rho_m/\rho_m$ in Eq. (84). Similarly the

terms on the r.h.s. of Eq. (85) can be neglected relative to those on the l.h.s., which leads to the following equation for matter perturbations:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\text{eff}}\rho_m\delta_m \simeq 0, \quad (93)$$

where the effective “cosmological” gravitational constant is given by

$$G_{\text{eff}} = \frac{1}{8\pi F} \frac{(k^2/a^2)(1+2Q^2)F + M^2}{(k^2/a^2)F + M^2}. \quad (94)$$

We can rewrite Eq. (93) by using the derivative with respect to N :

$$\frac{d^2\delta_m}{dN^2} + \left(\frac{1}{2} - \frac{3}{2}w_{\text{eff}}\right) \frac{d\delta_m}{dN} - \frac{3}{2}\Omega_m \frac{(k^2/a^2)(1+2Q^2)F + M^2}{(k^2/a^2)F + M^2} \delta_m \simeq 0. \quad (95)$$

We also define the effective gravitational potential $\Phi_{\text{eff}} \equiv (\Phi + \Psi)/2$, which is directly linked with the Integrated-Sachs-Wolfe (ISW) effect in CMB and the weak lensing in distant galaxies [13]. From Eq. (92) we obtain the relation

$$\Phi_{\text{eff}} \simeq -\frac{a^2}{2k^2} \frac{\rho_m}{F} \delta_m. \quad (96)$$

In order to confront models with weak lensing observations, it is convenient to introduce the anisotropic parameter η defined by, $\eta \equiv (\Phi - \Psi)/\Psi$ [45, 46]. From Eq. (92) we obtain

$$\eta \simeq \frac{4Q^2(k^2/a^2)F}{(k^2/a^2)(1-2Q^2)F + M^2}, \quad (97)$$

which vanishes in the limit $M^2/F \gg k^2/a^2$ but approaches a value $\eta \rightarrow 4Q^2/(1-2Q^2)$, in the limit $M^2/F \ll k^2/a^2$. We also introduce another parameter $\Sigma \equiv q(1 + \eta/2)$, where q is defined to be $(k^2/a^2)\Psi \equiv -(1/2)q\rho_m\delta_m$. We then have $\Sigma \simeq 1/F$, which shows that the effective potential can be written as $\Phi_{\text{eff}} \simeq -(a^2/2k^2)\rho_m\delta_m\Sigma$. Hence, unlike the case of the Einstein gravity the weak lensing potential in these scalar-tensor models of gravity are affected by the changes of Σ as well as δ_m .

During the matter era the field ϕ sits at the instantaneous minima characterized by the condition (70). This is analogous to the situation considered in subsection IV B where for models (41) the field value at the potential minimum and the mass squared M_ϕ^2 are given by Eqs. (60) and (61) respectively. Hence, we have the relations $\phi \propto \rho_m^{\frac{1}{p-1}}$ and $M^2 \propto M_\phi^2 \propto \rho_m^{\frac{2-p}{1-p}}$ during the matter-dominated epoch. The field ϕ can initially be heavy to satisfy the condition $M^2/F \gg k^2/a^2$ for the modes relevant to the galaxy power spectrum ($0.01h \text{ Mpc}^{-1} \lesssim k \lesssim 0.2h \text{ Mpc}^{-1}$). Depending upon the model parameters and the mode k , the mass squared M^2 can be smaller than k^2/a^2 during the matter era.

Next, let us consider the behavior of the oscillating mode. Using Eqs. (86) and (88) under the condition $k^2/a^2 \gg H^2$ the gravitational potentials for $\delta\rho_m = 0$ are expressed by $\delta\phi_{\text{osc}}$. Then from Eq. (90) the perturbation δR corresponding to the oscillating mode is given by

$$\delta R_{\text{osc}} \simeq 6Q \left(\delta\ddot{\phi}_{\text{osc}} + 3H\delta\dot{\phi}_{\text{osc}} + \frac{k^2}{a^2}\delta\phi_{\text{osc}} \right). \quad (98)$$

Substituting this relation in Eq. (89), we find

$$\delta\ddot{\phi}_{\text{osc}} + 3H\delta\dot{\phi}_{\text{osc}} + \left(\frac{k^2}{a^2} + \frac{M^2}{F} \right) \delta\phi_{\text{osc}} \simeq 0, \quad (99)$$

which is valid in the regimes $M^2 \gg \{R, \lambda^2 V\}$. Equation (99) clearly shows that the effective mass for the oscillating mode is positive even for $Q^2 > 1/6$.

In the following we shall confirm that as long as the oscillating mode does not initially dominate over the matter-induced mode, it remains subdominant throughout the cosmic history. We shall discuss the two cases: (i) $M^2/F \gg k^2/a^2$ and (ii) $M^2/F \ll k^2/a^2$, separately.

A. The case $M^2/F \gg k^2/a^2$

In this regime the matter perturbation equation (95) reduces to the standard one in Einstein gravity. During the matter era with, $w_{\text{eff}} \simeq 0$ and $\Omega_m \simeq 1$, we have the following solutions:

$$\delta_m \propto a \propto t^{2/3}, \quad \Phi_{\text{eff}} = \text{constant}. \quad (100)$$

For the model (41) the matter-induced mode of the field perturbation evolves as $\delta\phi_{\text{ind}} \propto \delta\rho_m/M^2 \propto t^{\frac{2(4-p)}{3(1-p)}}$. When the frequency $\omega_\phi = \sqrt{k^2/a^2 + M^2/F}$ changes adiabatically (i.e. $|\dot{\omega}_\phi/\omega_\phi^2| \ll 1$), the WKB solution to Eq. (99) is given by

$$\delta\phi_{\text{osc}} \propto a^{-3/2} \frac{1}{\sqrt{2\omega_\phi}} \cos\left(\int \omega_\phi dt\right). \quad (101)$$

For the model (41), in the regime $M^2/F \gg k^2/a^2$, this oscillating mode evolves as $\delta\phi_{\text{osc}} \propto t^{\frac{p}{2(1-p)}} \cos\left(ct^{-\frac{1}{1-p}}\right)$, where c is a constant.

Now since the background field ϕ during the matter era evolves as $\phi \propto t^{\frac{2}{1-p}}$, we find

$$\delta\phi/\phi \simeq c_1 t^{2/3} + c_2 t^{-\frac{4-p}{2(1-p)}} \cos\left(ct^{-\frac{1}{1-p}}\right). \quad (102)$$

This indicates that the matter-induced mode dominates over the oscillating mode with time. While the solution of the oscillating mode in Eq. (102) is valid only in the WKB regime ($|\dot{\omega}_\phi/\omega_\phi^2| \ll 1$), we have checked that $\delta\phi$ approaches a constant value with oscillations at the later stage in which the WKB approximation is violated. Hence, as long as the oscillating mode is not overproduced in the early universe, it remains sub-dominant relative to the matter-induced mode. Note that this property also holds during the radiation-dominated epoch.

B. The case $M^2/F \ll k^2/a^2$

In this regime the effective gravitational constant (94) is given by $G_{\text{eff}} = (1 + 2Q^2)/8\pi F$, which shows that the effect of modified gravity becomes important. From Eqs. (95) and (96) we obtain

$$\delta_m \propto t^{\frac{\sqrt{25+48Q^2}-1}{6}}, \quad \Phi_{\text{eff}} \propto t^{\frac{\sqrt{25+48Q^2}-5}{6}}, \quad (103)$$

which grow faster than the solutions given in Eq. (100). This leads to changes in the matter power spectrum of the LSS as well as in the ISW effect in the CMB.

The field perturbation $\delta\phi$ is the sum of the matter-induced mode given in Eq. (91) and the oscillating mode $\delta\phi_{\text{osc}}$ given in Eq. (99). Using the WKB solution (101) for the latter mode, we have

$$\delta\phi = c_1 t^{\frac{\sqrt{25+48Q^2}-5}{6}} + c_2 t^{-2/3} \cos(ct^{1/3}). \quad (104)$$

Since the frequency has a dependence $|\dot{\omega}_\phi/\omega_\phi^2| \simeq H \propto 1/t$, the WKB approximation tends to be accurate at late times. Equation (104) shows that the matter-induced mode dominates over the oscillating mode with time.

C. The matter power spectra

The models (41) have a heavy mass M which is much larger than H in the deep matter-dominated epoch, but which gradually decreases to become of the order of H around the present epoch. Depending on the modes k , the system crosses the point $M^2/F = k^2/a^2$ at $t = t_k$ during the matter era. In the context of $f(R)$ gravity this indeed happens for the modes relevant to the galaxy power spectrum [18, 19]. Since for the model (41) M evolves as $M \propto t^{-\frac{2-p}{1-p}}$ during the matter era, the time t_k has a scale-dependence given by $t_k \propto k^{-\frac{3(1-p)}{4-p}}$. When $t < t_k$, the evolution of δ_m is given by Eq. (100), but for $t > t_k$ its evolution changes to the form given by (103).

We define the growth rate of the matter perturbation to be

$$s \equiv \frac{\dot{\delta}_m}{H\delta_m}, \quad (105)$$

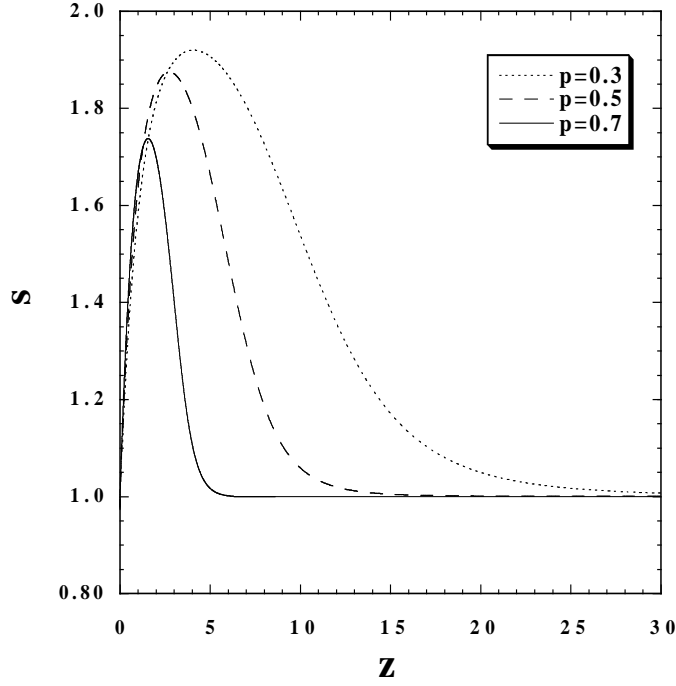


FIG. 4: The evolution of the growth rate s of matter perturbations in terms of the redshift z for $Q = 1.08$ and $k = 600a_0H_0$ with three different values of p . For smaller p the critical redshift z_k gets larger. The growth rate s reaches a maximum value and begins to decrease after the system enters the accelerated epoch. For smaller p the maximum value of s tends to approach the analytic value given in Eq. (106).

which is $s = 1$ in the regime $M^2/F \gg k^2/a^2$. After the system enters the regime $M^2/F \ll k^2/a^2$ during the matter-dominated epoch, we have

$$s = \frac{\sqrt{25 + 48Q^2} - 1}{4}. \quad (106)$$

During the matter era the mass squared is approximately given by

$$M^2 \simeq \frac{1-p}{(2^p p C)^{1/(1-p)}} Q^2 \left(\frac{\rho_m}{V_0} \right)^{\frac{2-p}{1-p}} V_0. \quad (107)$$

Using the relation $\rho_m = 3F_0\Omega_m^{(0)}H_0^2(1+z)^3$, we find that the critical redshift z_k at time t_k can be estimated as

$$z_k \simeq \left[\left(\frac{k}{a_0 H_0} \frac{1}{Q} \right)^{2(1-p)} \frac{2^p p C}{(1-p)^{1-p}} \frac{1}{(3F_0\Omega_m^{(0)})^{2-p}} \frac{V_0}{H_0^2} \right]^{\frac{1}{4-p}} - 1, \quad (108)$$

where a_0 is the present scale factor. The critical redshift increases for larger $k/(a_0 H_0)$. The matter power spectrum, in the linear regime, has been observed for the scales $0.01h \text{ Mpc}^{-1} \lesssim k \lesssim 0.2h \text{ Mpc}^{-1}$, which corresponds to $30a_0 H_0 \lesssim k \lesssim 600a_0 H_0$. In Fig. 4 we plot the evolution of the growth rate s for the mode $k = 600a_0 H_0$ and the coupling $Q = 1.08$ with three different values of p . We find that, in these cases, the critical redshift exists in the region $z_k \gtrsim 1$ and that z_k increases for smaller p . When $p = 0.7$ we have, $z_k = 3.9$, from Eq. (108), which is consistent with the numerical result in Fig. 4. The growth rate s reaches to a maximum value s_{max} and then begins to decrease around the end of the matter era.

McDonald *et al.* [47] derived the constraint $s = 1.46 \pm 0.49$ around the redshift, $z = 3$, from the measurement of the matter power spectrum from the Lyman- α forests. The more recent data reported by Viel and Haehnelt [48] in the redshift range $2 < z < 4$ show that even the value $s = 2$ can be allowed in some of the observations. The likelihood analysis using these data for the coupled quintessence scenario gives the constraint $s \lesssim 1.5$ [49]. If we use the criterion $s < 2$ for the analytic estimation (106), we obtain the bound $Q < 1.08$. Figure 4 shows that s_{max} is smaller than the analytic value $s = 2$ (which corresponds to $Q = 1.08$). When $p = 0.7$, for example, we have that $s_{\text{max}} = 1.74$. For

the values of p that are very close to 1, s_{max} can be smaller than 1.5. However these cases are hardly distinguishable from the Λ CDM model. In any case the current observational data on the growth rate s is not enough to place tight bounds on Q and p .

The growth of matter perturbations continues to the time t_Λ characterized by the condition $\ddot{a} = 0$. At time t_Λ the matter power spectrum $P_{\delta_m} = (k^3/2\pi^2)|\delta_m|^2$ shows a difference compared to the Λ CDM model given by

$$\frac{P_{\delta_m}(t_\Lambda)}{P_{\delta_m}^{\Lambda\text{CDM}}} = \left(\frac{t_\Lambda}{t_k}\right)^{2\left(\frac{\sqrt{25+48Q^2}-1}{6}-\frac{2}{3}\right)} \propto k^{\frac{(1-p)(\sqrt{25+48Q^2}-5)}{4-p}}. \quad (109)$$

The CMB power spectrum is also affected by the non-standard evolution of Φ_{eff} given in Eq. (103). This mainly happens for low multipoles because of the ISW effect. Since the smaller scale modes in CMB relevant to the galaxy power spectrum are hardly affected by this modification, there is a difference between the spectral indices of the matter power spectrum and of the CMB spectrum on the scales, $k > 0.01h \text{ Mpc}^{-1}$:

$$\Delta n(t_\Lambda) = \frac{(1-p)(\sqrt{25+48Q^2}-5)}{4-p}. \quad (110)$$

This reproduces the result in the $f(R)$ gravity derived in Ref. [15]. In Ref. [18] it was further shown that this analytic estimation agrees well with numerical results except for large values of p close to unity. This reflects the fact that for larger p the redshift $z = z_k$ at time $t = t_k$ gets smaller (being of the order of $z_k = \mathcal{O}(1)$) so the approximations used in deriving the solution (103), based on $w_{\text{eff}} = 0$ and $\Omega_m = 1$, break down. In Ref. [18] it was also found that the difference $\Delta n(t_0)$ integrated to the present epoch does not show significant difference compared to (110).

At present we do not have any observationally significant evidence for the presence of a difference between the spectral indices of the CMB and the matter power spectra [50]. In Fig. 5 we plot the constraints coming from the criterion, $\Delta n(t_\Lambda) < 0.05$. If $|Q|$ is smaller than 0.1, this condition is trivially satisfied. For larger $|Q|$ the constraints on the values of p tend to be stronger. In the $f(R)$ gravity we obtain the bound $p > 0.78$, which is stronger than the constraint coming from the violation of the equivalence principle. If we adopt the criterion $\Delta n(t_\Lambda) < 0.03$, the bound on p becomes tighter: $p > 0.87$. Meanwhile, if $|Q|$ is smaller than the order of 0.1, the EP constraint gives the tightest bound. If we use the criterion $s < 2$ for the analytic estimation (106) then the coupling $|Q|$ is bounded from above ($Q < 1.08$).

In Fig. 5 we show the allowed parameter space consistent with current observational and experimental constraints. The constraints coming from the ISW effect in the CMB due to the change in evolution of the gravitational potential do not provide tighter bounds compared to those shown in Fig. 5.

VII. CONCLUSIONS

We have considered a class of dark energy models based on scalar-tensor theories given by the action (13). In these theories, expressed in the Einstein frame, the scalar field ϕ is coupled to the non-relativistic matter with a constant coupling Q . The action (13) is equivalent to the Brans-Dicke theory with a field potential V , where the Brans-Dicke parameter ω_{BD} is related to the coupling Q via the relation $3 + 2\omega_{\text{BD}} = 1/2Q^2$. These theories include the $f(R)$ gravity theories and the quintessence models as special cases where the coupling is given by $Q = -1/\sqrt{6}$ (i.e., $\omega_{\text{BD}} = 0$) and $Q = 0$ (i.e., $\omega_{\text{BD}} \rightarrow \infty$), respectively.

We began by studying the background cosmological dynamics in a homogeneous and isotropic setting, without specifying the field potential $V(\phi)$ but under the assumption that the slope of the potential, $\lambda = -V_{,\phi}/V$, is constant. The varying λ case can also be studied by treating the fixed points as instantaneous ones. We found that for a range of values of the coupling constant $|Q|$ not much smaller than unity the matter era can be realized by the solution corresponding to the point (d) in Eq. (36) subject to the condition $\lambda/Q \gg 1$. Interestingly the presence of a non-zero coupling Q leads to a de-Sitter solution characterized by the condition $V_{,\phi} + QFR = 0$ (i.e., $\lambda = 4Q$), which can lead to late-time acceleration. (The condition for the stability of this de-Sitter solution is given by $d\lambda/d\phi < 0$ at the fixed point.)

In the absence of the scalar-field potential, solar-system tests constrain the coupling Q to have values in the range $|Q| < 2.5 \times 10^{-3}$. The presence of the potential, on the other hand, allows the LGC to be satisfied for larger values of $|Q|$, if the field is sufficiently heavy in the high-curvature region where gravity experiments are carried out. We found that even when $|Q|$ is of the order of 1, a thin-shell can form inside a spherically symmetric body such that the effective coupling $|Q_{\text{eff}}|$ defined in Eq. (55) becomes much smaller than 1.

We then considered a family of models given by the scalar-field potentials (41) which generalize the corresponding potential in the $f(R)$ theory, while at the same time satisfying the LGC for appropriate choices of the parameters.

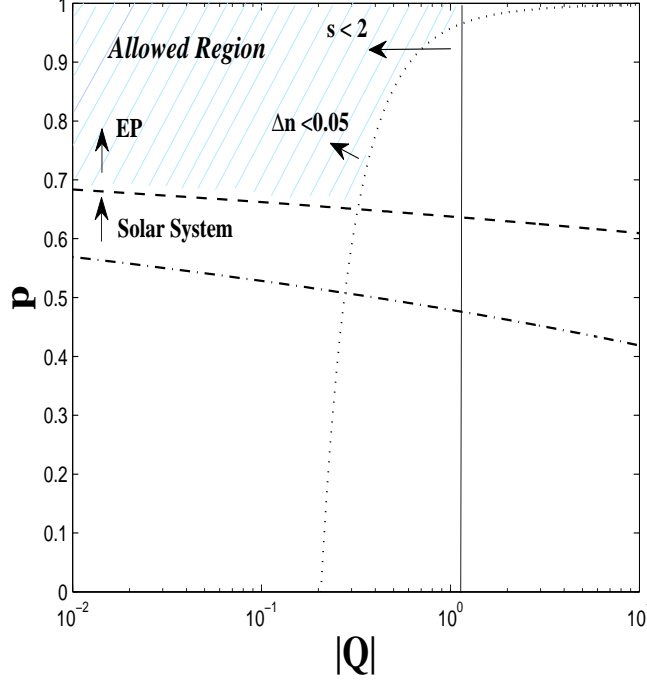


FIG. 5: The allowed region of the parameter space in the (p, Q) plane. We show the bounds coming from the conditions $\Delta n(t_A) < 0.05$ and $s < 2$ as well as the solar-system constraint (65) and the EP constraint (69).

In particular we found that as p approaches unity, the mass of the field ϕ becomes larger, thus allowing the LGC to be satisfied more easily [see Eq. (61)]. Using the constraints coming from solar system tests as well as compatibility with the equivalence principle, we obtained the bounds $p > 1 - 5/(9.6 - \log_{10}|Q|)$ and $p > 1 - 5/(13.8 - \log_{10}|Q|)$, respectively. In the $f(R)$ gravity, for example, these constraints correspond to $p > 0.50$ and $p > 0.65$ respectively.

During radiation/matter eras the field ϕ needs to be very close to 0 for the compatibility with LGC, which results in, $F = e^{-2Q\phi} \simeq 1$. Figure 5 summarizes the regions of the parameter space in the (p, Q) plane where the corresponding potentials lead to models compatible with the LGC.

For these models we found that the quantity F tends to increase from its present value as we go into the past, which results in the equation of state w_{DE} of dark energy becoming singular when $\Omega_m = F_0/F$. This behavior is similar to that found for $f(R)$ theories.

We also studied the evolution of density perturbations for these models in order to place constraints on the coupling Q as well as on the parameters of the field potential. In the deep matter era the mass M of the scalar field is sufficiently heavy to make these models compatible with LGC, but it gradually gets smaller as the Universe enters the accelerated epoch. For the models compatible with the galaxy power spectrum, there exists a “General Relativistic” phase during the matter era characterized by the condition $M^2/F \gg k^2/a^2$. At this stage the matter perturbation δ_m and the effective gravitational potential Φ_{eff} evolve as $\delta_m \propto t^{2/3}$ and $\Phi_{\text{eff}} = \text{constant}$, respectively, as in the case of Einstein gravity. Around the end of the matter-dominated epoch, the deviation from the Einstein gravity can be seen once M^2/F becomes smaller than k^2/a^2 . The evolution of perturbations during this “scalar-tensor” regime is given by Eqs. (103). Under the criterion $s = \dot{\delta}_m/H\delta_m < 2$ of the growth rate of matter perturbations with the use of the analytic estimation (106), we obtain the bound $Q < 1.08$. The difference Δn of the spectral indices of the CMB and the matter power spectra gives rise to another constraint on the model parameter p and the coupling Q .

Figure 5 illustrates the bounds derived from the conditions $\Delta n < 0.05$ and $s < 2$, as well as those from local gravity constraints. The models with p close to 1 satisfy all these requirements. It will be certainly of interest to place more stringent constraints on the values of p and Q by using the recent data of matter power spectrum, CMB and Lyman alpha forest. Moreover, the future survey of weak lensing may find some evidence of an anisotropic stress between gravitational potentials Φ and Ψ , which can be a powerful tool to distinguish modified gravity models from the Λ CDM cosmology.

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APPENDIX A: STABILITY ANALYSIS

In this Appendix, we briefly summarize the results of the stability analysis which are necessary to obtain the background cosmological dynamics discussed in Sec. III.

1. Stability of the fixed points

When λ is a constant, one can analyze the stability of the critical points (a)-(e) [i.e., Eqs. (32)-(37)] by considering small perturbations δx_1 and δx_2 around them [2]. We write the equations for perturbations in the form

$$\frac{d}{dN} \begin{pmatrix} \delta x_1 \\ \delta x_2 \end{pmatrix} = \mathcal{M} \begin{pmatrix} \delta x_1 \\ \delta x_2 \end{pmatrix}, \quad (\text{A1})$$

and derive eigenvalues μ_1 and μ_2 of the matrix \mathcal{M} to assess the stability of fixed points. They are given by

• (a)

$$\mu_1 = -\frac{3-2Q^2}{2(1-2Q^2)}, \quad \mu_2 = \frac{3+2Q\lambda-6Q^2}{2(1-2Q^2)}. \quad (\text{A2})$$

• (b1)

$$\mu_1 = \frac{3(\sqrt{6}+4Q-\lambda)}{\sqrt{6}+6Q}, \quad \mu_2 = \frac{3+\sqrt{6}Q}{1+\sqrt{6}Q}. \quad (\text{A3})$$

• (b2)

$$\mu_1 = \frac{3(\sqrt{6}-4Q+\lambda)}{\sqrt{6}-6Q}, \quad \mu_2 = \frac{3-\sqrt{6}Q}{1-\sqrt{6}Q}. \quad (\text{A4})$$

• (c)

$$\mu_1 = -\frac{6-\lambda^2+8Q\lambda-16Q^2}{2(1-4Q^2+Q\lambda)}, \quad \mu_2 = -\frac{3-\lambda^2+7Q\lambda-12Q^2}{1-4Q^2+Q\lambda}. \quad (\text{A5})$$

• (d)

$$\mu_{1,2} = \frac{3(2Q-\lambda)}{4\lambda} \left[1 \pm \sqrt{1 + \frac{8(6Q^2-2Q\lambda-3)(12Q^2+\lambda^2-7Q\lambda-3)}{3(2Q-\lambda)^2}} \right]. \quad (\text{A6})$$

• (e)

$$\mu_1 = \mu_2 = -3. \quad (\text{A7})$$

2. Stability of the de-Sitter point for the variable λ

While the point (e) is stable for constant λ , it is not obvious that this property also holds for a varying λ . In what follows we shall discuss the stability of the de-Sitter point.

It is convenient to consider the variable $\lambda(\phi)$ as a function of $F(\phi)$, i.e., $\lambda = \lambda(F)$. We define a variable, $x_4 \equiv F$, that satisfies the following equation

$$\frac{dx_4}{dN} = -2\sqrt{6}Qx_1x_4, \quad (\text{A8})$$

where the r.h.s. vanishes at the de-Sitter point (e). Considering the 3×3 matrix for perturbations δx_1 , δx_2 and δx_4 around the point (e), we obtain the eigenvalues

$$\mu_1 = -3, \quad \mu_{2,3} = -\frac{3}{2} \left[1 \pm \sqrt{1 - \frac{8}{3}F_1 Q \frac{d\lambda}{dF}(F_1)} \right], \quad (\text{A9})$$

where, $F_1 \equiv F(\phi_1)$ is the value of F at the de-Sitter point with the field value ϕ_1 . Since, $F_1 > 0$, we find that the de-Sitter point is stable for

$$Q \frac{d\lambda}{dF}(F_1) > 0, \quad \text{i.e.,} \quad \frac{d\lambda}{d\phi}(\phi_1) < 0. \quad (\text{A10})$$

We checked that this agrees with the stability condition derived in Refs. [51] by considering metric perturbations about the de-Sitter point.

In $f(R)$ gravity this condition translates into $d\lambda/dF < 0$. Since in this case, $F = e^{2\phi/\sqrt{6}} = df/dR$ and $V = (RF - f)/2$, we have $\lambda = -Rf_{,R}/\sqrt{6}V$. Then, together with the fact that $Rf_{,R} = 2f$ holds for the de-Sitter point, the condition, $d\lambda/dF < 0$, is equivalent to $R < f_{,R}/f_{,RR}$. For positive R this gives

$$0 < \frac{Rf_{,RR}}{f_{,R}} < 1, \quad (\text{A11})$$

which agrees with the stability condition for the de-Sitter point derived in Ref. [11].

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